Journal of Fluids and Structures (1998) **12,** 427–444
Article No. fl970150

VORTEX-INDUCED VIBRATION AND DAMPING OF THERMOWELLS

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(Received 12 December 1996 and in revised form 17 October 1997)

Thermowells are installed in process piping to protect fluid temperature measuring instruments from the fluid flow. The thermowells are subject to vortex-induced vibrations by the process fluid. The resonant response is limited by damping. Measurements of thermowell vibration damping were made and used to establish criteria for acceptable design under vortex-induced vibration. Comparison is made with the damping of steel stacks and heat exchanger tubes.

1. INTRODUCTION

Thermowells are metal cylinders used to house and protect thermocouples in piping. As shown in Figure 1, a thermowell consists of a closed-ended hollow cantilever that is welded to a heavy flange that is in turn bolted through a gasket to a mating flange on the exterior of the pipe. Thermocouples are inserted into the thermowell to measure the fluid temperature without contacting the fluid. Typical thermowell unsupported lengths (U) are between 10 and 20 in (250 and 500 mm) and typical diameters are between 0·75 and 1·5 in (19 and 38 mm). Thermowells are generally fabricated from carbon steel, nickel alloys, or 300 series austenitic stainless steel. Most thermowells have a constant outside diameter, but some thermowells taper from the flange to the tip. The inner bore is held constant. Figure 2 is a photograph of a thermowell.

Thermowells are used to measure the temperature of fluids ranging from high-density boiler feed water and liquid hydrocarbons to low-density gases including stack gas, acid gas, sulfur vapor, steam, and tail gas. Vertices are shed in the fluid flow. There is potential resonance between the first or second cantilever modes of the thermowells with vortex shedding. The ASME Performance Test Codes Supplement on Instruments (1974) endorses design such that the vortex shedding frequency does not exceed 0·8 times the natural frequency, eliminating the possibility of resonance. This stiffness is difficult to to achieve in many designs.

Recent research indicates that the resonant response to vortex shedding is limited by mass and damping (Blevins 1990; Sarpkaya 1979). Tap tests, also called ping tests, were

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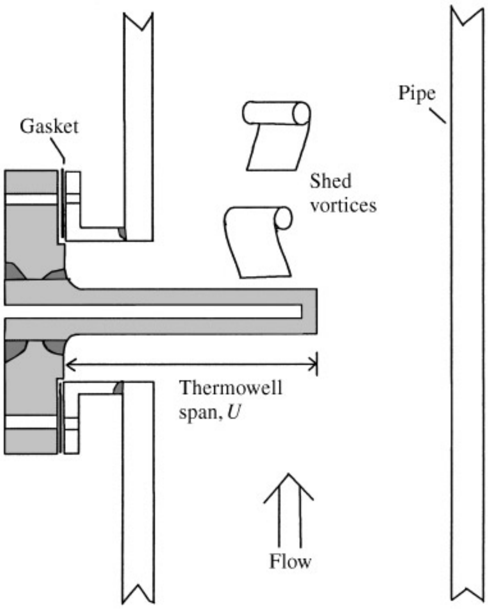

Figure 1. Typical thermowell installation. The instrument that fits inside the thermowell is not shown.

made to measure thermowell vibration damping. Eleven thermowells were tested with various gasket and instrument configurations and 94 measurements of damping were made. Thermowell damping factors between 0·0005 and 0·002 (0·05–0·2% of critical damping) are recommended for design.

2. VORTEX-INDUCED VIBRATION ANALYSIS

Vortices are shed from a circular cylinder in a flow to create the vortex wake called a vortex street. The frequency of vortex shedding is given in terms of a dimensionless Strouhal number S,

$$f_s = SV/D, \tag{1}$$

in Hz; V is the mean flow velocity normal to the cylinder (note that the symbol U is reserved for thermowell free length, Figure 1), D is the cylinder outside diameter, and S is the dimensionless Strouhal number, which is approximately $S = 0{\cdot}2$ in the range of Reynolds numbers of interest in most petrochemical piping. The shedding vortices impose a lift (normal to free stream) force that oscillates at the shedding frequency [equation (1)]. The oscillating life force, F_t, on the cylinder per unit length can be approximately expressed in terms of a lift coefficient C_L times the dynamic head $(\frac{1}{2}\rho V^2)$ of the stream,

$$F_L = \tfrac{1}{2}\rho V^2 D C_L \sin(2\pi f_s t). \tag{2}$$

When the shedding frequency approaches a natural frequency of the cylinder, a condition called resonance, the cylinder begins to respond to the shedding lift forces, and the

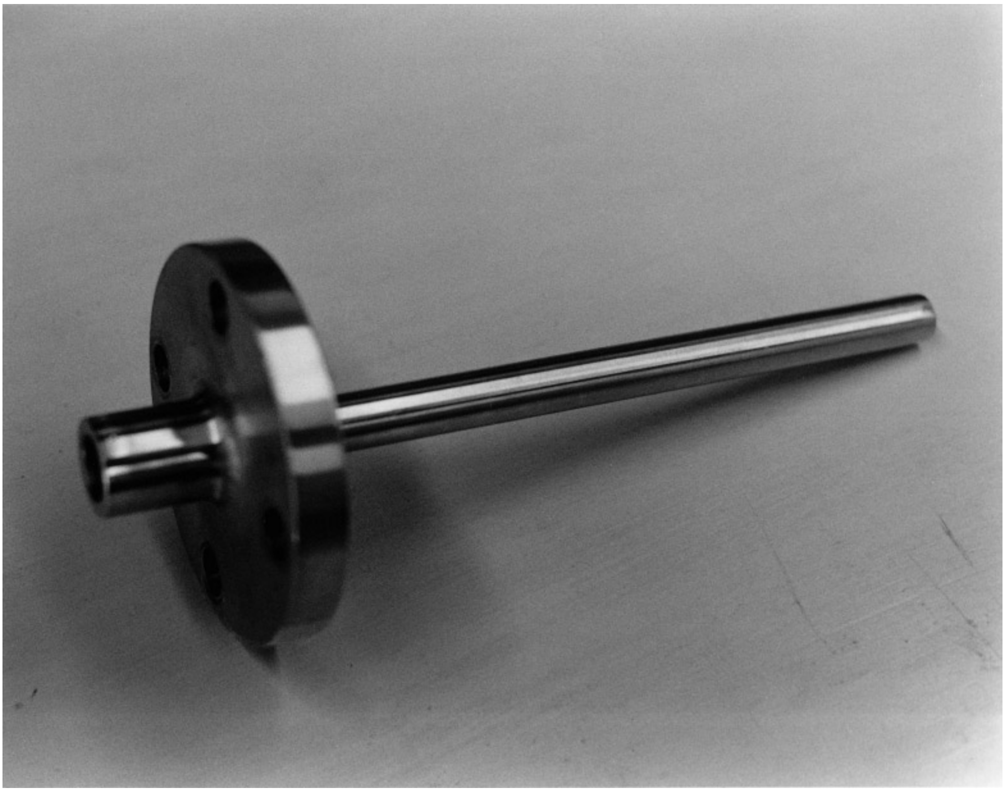

Figure 2. Photograph of a test thermowell.

vortex-shedding frequency can suddenly shift to the cylinder natural frequency, a phenomenon called lock-in or synchronization. At resonance, the cylinder motion organizes the shedding, and the resultant coherent forces can result in damaging vibration.

The natural frequency of a straight uniform thermowell is (Weaver *et al.* 1990; Blevins 1979)

$$f_n = \frac{\lambda_n^2}{2\pi U^2}\sqrt{\frac{EI}{m}}, \qquad n = 1, 2, 3 \tag{3}$$

The dimensionless natural frequency parameter of a cantilever is $\lambda_1 = 1.875$ for first-mode and $\lambda_2 = 4{\cdot}694$ for second-mode vibrations. E is the modulus of elasticity, I the moment of inertia, and m the mass per unit length including external added mass and internal mass. At resonance, the shedding frequency f_s of equation (1) equals the natural frequency f_n of equation (3), $f_s = f_n$, and the response is limited by damping of the thermowell.

If one uses a simple spring-mass model of thermowell vibration in a single mode with spring constant k, the amplitude (zero-to-peak) of the resonant vibration response to the vortex force of equation (2) is inversely proportional to damping (Blevins 1990, p. 62):

$$\frac{A_y}{D} = \frac{\rho V^2 C_L}{4k\zeta} = \frac{C_L}{4\pi S^2 \delta_r}; \tag{4}$$

ζ is the damping factor, also called the damping ratio or fraction of critical damping. Equation (1) has been substituted on the right-hand side of this equation to eliminate the velocity term, and a dimensionless mass-damping parameter, called reduced damping, has been introduced:

$$\delta_r = \frac{2m(2\pi\zeta)}{\rho D^2}; \tag{5}$$

ρ is the fluid density, and m is the mass per unit length of the cylinder including added mass and any internal mass. It is generally accepted that the value $C_L = 1$ is conservative (ASME Boiler and Pressure Vessel Code 1992; Appendix N-1300), but it is probably too conservative for small amplitudes because of nonlinear fluid phenomena and finite correlation along the span. Recent work indicates that a nonlinear model is more accurate (Sarpkaya 1979; Blevins 1990, p. 71), but here for simplified analysis the linear model, equation (4) with $C_L = 0{\cdot}35$ is used.

Vortex-induced vibration of the thermowell excites one of the natural modes of the thermowell. The thermowell responds at its natural frequency in that mode,

$$Y(x, t) = A_y y(x) \sin(2\pi f_n t + \phi). \tag{6}$$

$Y(x, t)$ is the time-history lateral displacement of the thermowell, A_y the zero-to-peak modal amplitude, $y(x)$ the mode shape in the mode of interest, and f_n the natural frequency in Hz. The vibration induces cyclic strain in the thermowell. The maximum bending strain is at the base of the cantilever. The strain at the base of a straight uniform cantilever due to vortex shedding is

$$\varepsilon = \frac{\sigma}{E} = \frac{D\partial^2 Y}{2\partial x^3} \simeq \frac{\lambda^2 C_L D^2}{8\pi S^2 \delta_r U^2}. \tag{7}$$

The right-hand side of this expression applies to the straight cantilever thermowell of outside diameter D and length U, and it was obtained by substituting equations (4) and (6) into the classical formulation for bending stress in a uniform beam (Weaver *et al.*, 1990, p. 417). Equation (7) shows that the resonant strain is inversely proportional to damping.

For a typical thermowell length to diameter ratio of $U/D = 10$, a Strouhal number $S = 0{\cdot}2$, and a lift coefficient of $C_L = 0{\cdot}35$, it is necessary to have a reduced damping [equation (5)] $\delta_r = 15$ or higher to limit the first-mode resontant limit tip-amplitude to 5% of the diameter [equation (3)] and limit strain [equation (5)] to 0·0008 (approximately 22 ksi, 15·7 hbar, peak oscillatory stress), which is a typical high cycle fatigue allowable for austenitic stainless steel (ASME Boiler and Pressure Vessel Code 1995). Thus, we have two criteria for avoiding vortex-induced vibration damage to thermowells: (i) avoid resonance by stiffening the thermowell: $0{\cdot}8 f_n > f_s$, or (ii) limit fatigue strain at resonance with damping: $\delta_r > 15$.

Limiting strain at resonance by damping requires knowledge of the vibration damping factor of the thermowell vibration.

3. ASME ADVICE ON DAMPING

Vortex-induced vibration is treated in three sections of the ASME standards and code:

(a) ASME Boiler and Pressure Vessel Code. Section III, Division 1, (Nuclear) Appendix N-1300, 1992. Flow-Induced Vibration of Tubes and Tube Banks;

(b) ASME STS-1-1992. Steel Stacks. Para 5.3.2 (c), 1993;
(c) ASME Performance Test Codes. PTC 19.3-1974 (R1986) Temperature Measurement. Chapter 1, Para. 14, 1974.

Appendix N-1300, which was developed for nuclear heat exchangers, recommends damping ratios from 0·1% in gas to about 1% in steam or water for small amplitude vibration of tubes passing through oversized holes in support plates.

If the shedding frequency is 70% of the natural frequency or greater, then the ASME Steel Stacks standard STS-1-1992 considers the possibility of resonance and the effect of damping on the resonant response. STS-1-192 defines a mass-damping parameter m' which is $1/4\pi$ times the present reduced damping, equation (5):

$$m' = \frac{m\beta}{\phi d^2} = \frac{m\zeta}{\rho D^2} = \frac{1}{4\pi}\,\delta_r. \tag{8}$$

The left-hand portion of this equation uses the STS-1 notation and the right-hand side uses the present notation. STS-1 states that if $m' < 0{\cdot}4$ ($\delta_r < 5{\cdot}02$), then the resonant amplitude is damaging, if $0{\cdot}4 < m' < 0{\cdot}8$, that is $5{\cdot}02 < \delta_r < 10{\cdot}04$, then large amplitudes are possible, and if $m' > 0{\cdot}8$ ($\delta_r > 10{\cdot}04$), then a cross-wind response is less significant. This is consistent with the proposal of the previous section that $\delta_r > 15$ avoids damaging thermowell vibrations.

Values for the damping factors of stacks are recommended in STS-1, as listed in Table 1.

ASME Performance Test Codes PTC 19.3-1974 (R1986) gives no guidelines for damping. Literature searches revealed no data for damping of thermowells. Data from the piping literature (Blevins 1990, p. 327) gives the damping factors tabulated in Table 2.

Since thermowell construction is a welded construction, these last two sets of data suggest that thermowell damping factors are of the order of $\zeta = 0{\cdot}002$.

TABLE 1
Damping factors recommended in STS-1

Welded stack	Damping factor ζ		
	Low	Average	High
Unlined	0·0016	0·004	0·006
Lined	0·0032	0·0067	0·010

TABLE 2
Damping factors from the piping literature

Piping	Damping factor ζ		
	Minimum	75% above	Average
Power plant piping	0·002	0·019	0·0399
Heat exchanger tubing, air	0·002	0·0079	0·0169
Heat exchanger tubing, H_2O	0·0051	0·010	0·0196

4. DAMPING TESTS

4.1. Description of the Tests

The damping of thermowells was measured experimentally by installing thermowells in a heavy steel fixture, tapping the thermowell with an instrumented hammer, and then determining the damping from the free decay of the thermowell vibrations. All tests were made in air at room temperature.

An inventory of test thermowells is given in Table 3. An inventory of gaskets is listed in Table 4. The thermocouple instruments are listed in Table 5. The tests were made with thermowell ranging from $U = 7$ to 16 in (178–406 mm) and with both tapered and straight thermowells, in order to determine if there were any trends in damping with thermowell length or taper. Similarly, both fiber and metallic gaskets were used. For a few tests, the instruments were inserted into the thermowells to determine if these changed damping. Because of the very large potential matrix of the tests, the 10 in (250 mm), 150 lb class thermowell with fiber gasket was chosen as the standard, and variations were made about this standard configuration.

There were two fixture plates, one with 0·5 in (12·7 mm) studs for 150 lb thermowells and one with 0·75 in (19 mm) studs for 300 and 600 lb thermowells. The fixtures were fabricated from 0·75 in (19 mm) thick steel plate with standard weight pipe welded to the plate to provide a nozzle configuration typically used for thermowell installations. Figure 3 shows the thermowell bolted to fixture plates, which were in turn bolted to 1 in (25·4 mm) thick

Table 3
Thermowell inventory

No.	Description	Straight or tapered	Bore ID, (in)	Length U, (in)	Root dia, (in)	Tip dia, (in)
1	$1\frac{1}{2}$ in 150 #	Straight	1/4	7	0·752	0·752
2	$1\frac{1}{2}$ in 150 #	Straight	1/4	10	0·750	0·750
3	$1\frac{1}{2}$ in 300 R	Straight	1/4	10	0·752	0·752
4	$1\frac{1}{2}$ in 600 #	Straight	1/4	10	0·750	0·750
5	$1\frac{1}{2}$ in 150 #	Straight	3/8	10	0·878	0·878
6	$1\frac{1}{2}$ in 150 #	Tapered	1/4	10	0·867	0·633
7	$1\frac{1}{2}$ in 150 #	Straight	1/4	13	0·755	0·755
8	$1\frac{1}{2}$ in 150 #	Straight	1/4	16	0·760	0·760
9	$1\frac{1}{2}$ in 300 #	Straight	3/8	10	0·879	0·879
10	$1\frac{1}{2}$ in 600 #	Straight	3/8	8	0·878	0·878
11	$1\frac{1}{2}$ in 150 #	Tapered	3/8	9	1·0	0·769

Table 4
Gasket inventory

No.	Gasket	Description	Torque, fi-lbs
1	HT F1684 R20/316	Brass ring	n/a
2	GR $1\frac{1}{2}$ CL300 1/16 FR	Fiber gasket	65
3	GR $1\frac{1}{2}$ CL150 1/16 FR	Fibre gasket	20
4	LS-CG304L/Flexicarb $1\frac{1}{2}$-150	Metallic	30
5	CG304L/Flexicarb $1\frac{1}{2}$-3/4/600	Metallic	100

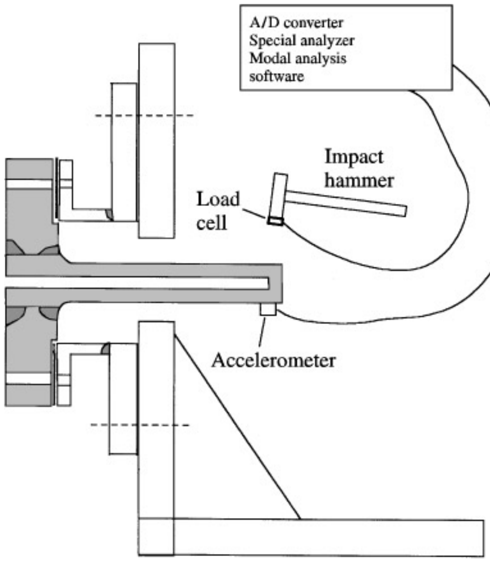

Figure 3. Test fixture and data system.

TABLE 5
Temperature measuring instruments inserted into thermowells

No.	Description	Length (in)
1	Bi-metal dial thermometer	$11\frac{3}{8}$
2	Electrical thermocouple with cap	12
3	Spring loaded thermocouple	$11\frac{3}{8}$
4	$\frac{1}{4}$ in (6·4 mm) diameter steel rod	$11\frac{3}{8}$

TABLE 6
Installation torques for the thermowell studs

Gasket type	0·5 in (12·7 mm) studs	0·75 in (19 mm) studs
Flexitallic gasket	30 ft lbs (41 N m)	100 ft lbs (136 N m)
Fiber gasket	20 ft lbs (27 N m)	65 ft lbs (88 N m)

steel right angle fittings, which were in turn bolted to the steel floor. The fixturing and its foundation were effectively rigid.

The thermowells were fabricated from 304 or 316 stainless steel. For the 150 lb class thermowells, the flanges were 0·675 in (17 mm) thick and 5 in (127 mm) in diameter and the seal was seated against against a 2·88 in (73·2 mm) in diameter gasket face raised 0·1 in (2·5 mm) off the flange. For the 300 lb class thermowells, the flange was 6·1 in (155 mm) in diameter, 0·74 in (18·8 mm) thick, the raised gasket face was 2·88 in (73·2 mm) in diameter and raised 0·1 in (2·5 mm) off the flange. For the 600 lb class thermowell, the flange was 0·838 in (21·3 mm) thick [0·965 in (24·5 mm) thick for thermowell 4] and the raised gasket face was raised 0·285 in (7·24 mm). Installation torques for the four studs that held the thermowells to the flange were as given in Table 6.

Test were made as follows. The test thermowell was mounted in the fixture and the studs were torqued to specifications. An instrument was installed if required. A miniature accelerometer (weight 0·3 g) was attached to the free end of the thermowell. The free end of the thermowell was impacted with a modal hammer as shown in Figure 3. The impulse to the thermowell and the response of the thermowell were recorded. The analog signal was sent to a two-channel analog-to-digital converter and then to a workstation where SDRC Ideas software processed the time series using fast Fourier transforms to compute the spectra of both signals and form the transfer function between the input (hammer) and response (accelerometer). Five hits were made and the average of the five hits was used to determine damping. Figure 4 shows a time history of the tip accelerometer for thermowell test 5 (Table 7). Note the initial impulse and then the ring down to decay of the fundamental mode. The rapid initial decay indicates that the damping of the thermowell increases with amplitude.

Because thermowell damping varies with amplitude, it is most useful to have damping factors measured at the amplitude of interest. The amplitude of interest for flow-induced vibration is the amplitude at the onset of high cycle fatigue damage. As noted in Section 2, a typical thermowell experiences high cycle fatigue damage for tip amplitudes about 5% of the root diameter, about 0·035 in (1 mm). It proved difficult to obtain this high excitation with the modal test equipment, and the majority of measurements reported here are the damping with tip amplitude of the order of 0·010 to 0·020 in (0·25 to 0·5 mm).

Figure 5 shows the spectrum of tip acceleration for thermowell test 2 (Table 7). The three peaks are the first natural modes of the thermowell. There are two methods of estimating damping from these plots. The first is to use the envelope of history decay. The damping factor is computed from the rate of decay,

$$\zeta = 0{\cdot}1103/N, \tag{9}$$

N being the number of vibration cycles required for the vibration amplitude to decay by a factor of 2. The more rapid the decay, the larger the damping. The slow decay in Figure 4

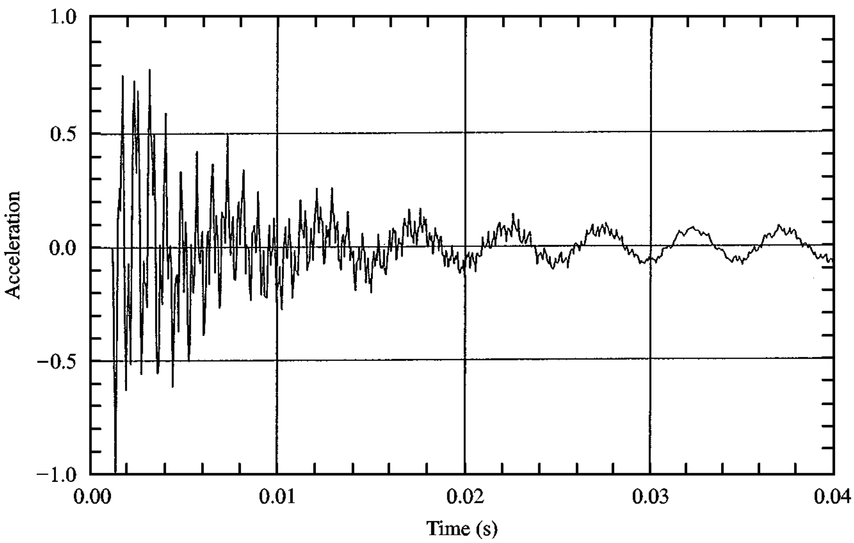

Figure 4. Time history response of the thermowell tip after impulse.

TABLE 7
Measured thermowell damping

	Thermowell					Mode 1			Mode 2		Mode 3		
Test No.	No.	Length (in)	Gakset type	Torque (ft lb)	Thermo-couple	Freq. (Hz)	Modal damping factor	Transient damping factor	Freq. (Hz)	Model damping factor	Freq. (Hz)	Modal damping factor	Peak Disp. (in)
1	1	10	None	30	None	207·9	0·00046	0·00040	1287·0	0·00370	3565	0·00090	
2	2	10	3-fiber	20	None	206·4	0·00049	0·00050	1268·1	0·00291	3643·9	0·00488	0·026
3	2	10	4-metallic	30	None	206·9	0·00050	0·00042	1280·1	0·00336	3554·7	0·00075	
4	2	10	3-fiber	20	3-stand.	200·5	0·00046	0·00038	1263·7	0·01697	3634·6	0·00580	
5	2	10	3-fiber	20	1-dial	202·9	0·00360	0·00116	1252·4	0·01578	3648·2	0·00471	
6	2	10	3-fiber	20	2-electric	199·2	0·00060	0·00044	1250·1	0·00774			
7	5	10	3-fiber	20	None	238·6	0·00051	0·00053	1452·6	0·00285			
8	5	10	3-fiber	20	1-dial	244·5	0·01356	0·01873	1452·9	0·00458	4165·9	0·00183	
9	5	10	3-fiber	20	S-stand.	239·9	0·00337	0·00324	1455·9	0·00323	4166·1	0·00220	
10	6	10	3-fiber	20	None	278·3	0·00059	0·00091	1366·6	0·00293	3739·5	0·00985	
11	1	7	3-fiber	20	None	407·1	0·00053	0·00075	2249·8	0·00581			
12	7	13	3-fiber	20	None	124·0	0·00064	0·00042	769·3	0·00068	2054·7	0·00764	
13	8	16	3-fiber	20	None	83·2	0·00058	0·00033	519·7	0·00052	1423·9	0·00136	0·004
14	11	10	3-fiber	20	None	377·9	0·00067	0·00056	1832·4	0·00440	5178·6	0·00035	
15	3	10	None	100	None	205·6	0·00046	0·00052	1273·3	0·00320	3500·6	0·00040	0·043
16	3	10	2-fiber	65	None	205·2	0·00043	0·00076	1264·9	0·00346	3587·1	0·00387	
17	3	10	5-metallic	100	None	205·2	0·00038	0·00025	1268·1	0·00365	3492·1	0·00053	
18	4	10	None	100	None	207·3	0·00026	0·00030	1281·1	0·00358	3522·7	0·00045	0·011
19	4	10	5-metallic	100	None	206·9	0·00034	0·00024	1274·8	0·00415	3516·1	0·00075	
21	4	10	5-metallic	100	1-dial	203·0	0·00273	0·00194	1277·9	0·00971	3512·6	0·00480	
22	4	10	5-metallic	100	3-stand	200·9	0·00208	0·00104	1269·9	0·01028	3513·5	0·00204	
23	6	10	3-fiber	20	1-dial	274·6	0·00700	0·02340	1364·2	0·01742	3557·7	0·00720	
24	2	10	3-fiber	20	Loose rod	208·5	0·00544	0·01656	1272·5	0·00903			
25	2	10	3-fiber	20	Curved rod	208·3	0·02050	0·01939	1270·2	0·01276			
26	2	10	3-fiber	20	Loose rod	209·5	0·04440	0·06213	1269·3	0·01278			

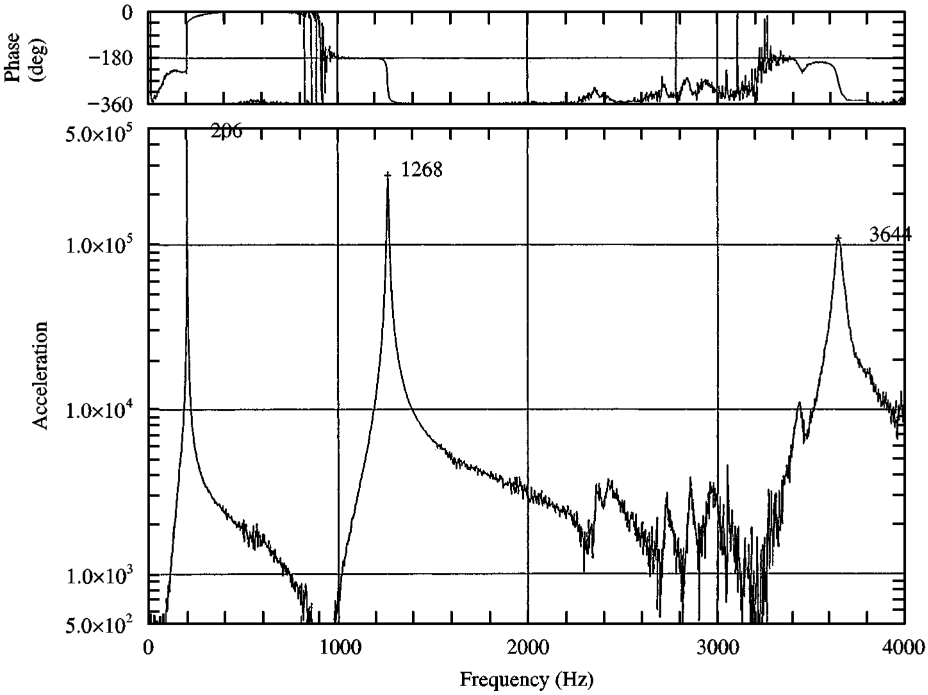

Figure 5. Spectrum of tip acceleration for thermowell test 2.

is indicative of low damping. This works best for the fundamental mode, where vibration cycles are most easily counted. The interval over which the cycles were counted was chosen to be part of the initial decay to characterize damping during relatively high amplitude vibration.

A second method for measuring damping from spectra is the bandwidth method. The damping factor is

$$\zeta = \Delta f/(2f_n), \tag{10}$$

where f_n is the natural frequency of a mode (peak in the spectra) and Δf is the width of that peak at a distance a factor of $2^{1/2}$ below the spectral peak. This method was used for the fundamental and higher modes.

4.2. Test Results

The test results for natural frequency and damping are summarized in Table 7, There is a considerable range of damping factors, from 0·000242 to 0·06213, with an average of 0·00526. The damping data is plotted as a function of frequency in Figure 6. There is no clear trend in terms of frequency, mode, gasket material, thermowell length, or taper in the damping data. It is clear that the data obtained with a thermocouple installed was higher average damping than the data obtained without a thermocouple installed. Unfortunately, one cannot guarantee that a thermocouple or thermometer will always be installed in a thermowell.

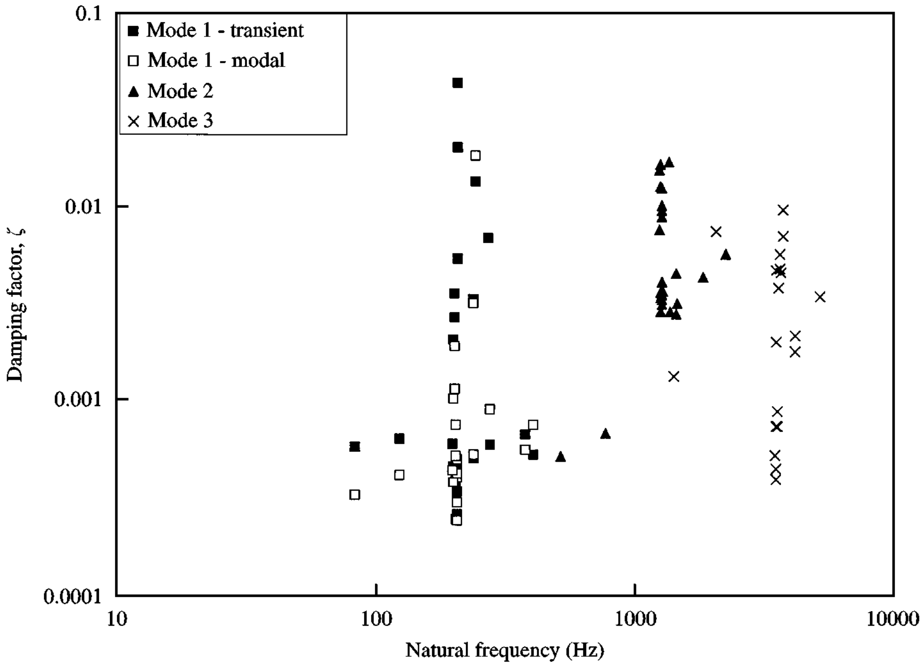

Figure 6. Thermowell damping factor as a function of natural frequency.

The scatter in damping data suggests that it should be treated statistically. As shown in Figure 7, 75% of the damping exceeds 0·0005 (0·05% damping). The median value (50% above and 50% below) is 0·002 (0·2% damping). Based on this, values of damping between $\zeta = 0{\cdot}0005$ and $\zeta = 0{\cdot}002$ are recommended for thermowell design.

The natural modes of the thermowells were measured for the 16 in (406 mm) long thermowell. These are shown in Figure 8. The observed mode shapes closely match the predicted classical (clamped-free) cantilever mode shapes. Similarly, the measured frequencies of these modes closely match the classical natural frequency predictions, as may be seen in Table 8. Thus, classical methods for natural frequency prediction work well with thermowells.

5. DAMPING OF HEAT EXCHANGER TUBES WITH ROLLED-IN ENDS

In this section results are presented for damping of heat exchanger tubes with rolled-in ends for comparison of the thermowell test results. The first example is a tube and shell-fossil-fired boiler that is designed to produce superheated steam. As shown in Figure 9, the boiler consists of a lower water drum connected by 2 in (5 cm) diameter, 0·134 in (3·4 mm) wall steel boiler tubes to an upper steam drum. The boiler tubes are rolled into the drums. The tubes are single span tubes, 8–11 ft (2·4–3·3 m) in length. They are supported only at the drums; there are no intermediate supports. Tap tests, using equipment similar to that described in the previous section, were used to measure the damping with the unit shut down. The results for natural frequency and damping in the fundamental mode are given in Table 9.

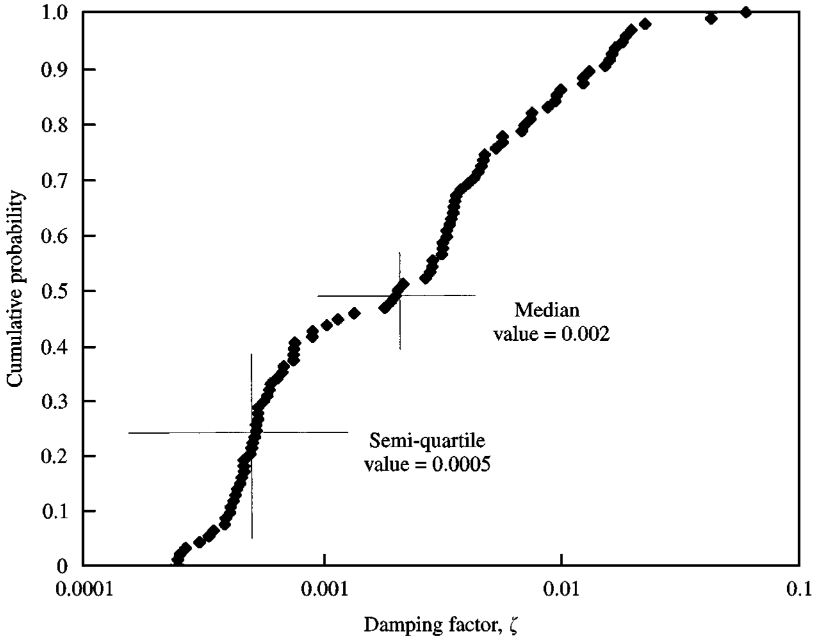

Figure 7. Cumulative probability of thermowell damping.

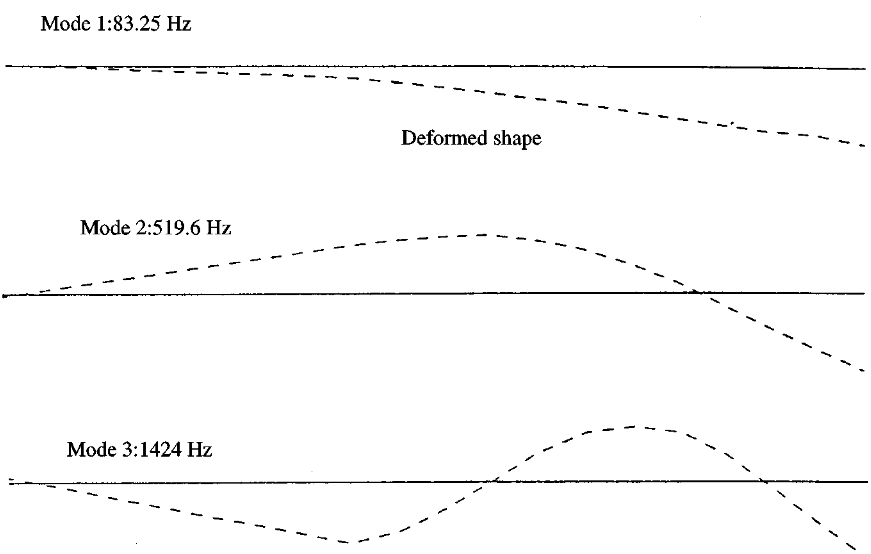

Figure 8. Mode shapes of the 16 in (406 mm) thermowell.

Also given in Table 9 are the results of tap tests made on 0·75 in (19 mm) outside diameter copper tubes, 0·070 in (1·8 mm) wall with fins. These multi-span tubes were rolled-in at the tube sheet and at the intermediate $\frac{3}{8}$ in (9·5 mm) thick baffle plates. Frequency analyses of the results show that the rolling-in creates a clamped boundary condition, so the individual

TABLE 8
Measured and predicted thermowell frequencies

	Mode 1	Mode 2	Mode 3
Measured frequency (Hz)	83·2	519·7	1423·9
Predicted frequency via equation (3) (Hz)	84·7	531·0	1486·7

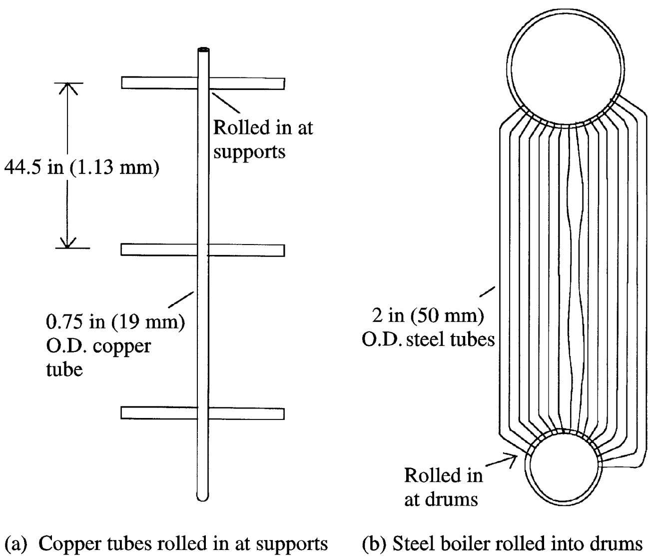

Figure 9. Heat exchanger tubing that has been rolled-in at the tube supports.

tube spans respond at the same frequency as a clamped–clamped tube. The distance between rolled-in supports was 44·5 in (1·13 m). Like the boiler tubes, these tubes were tap tested in air with the unit shut down. The first mode natural frequency was approximately 41 Hz and the second-mode natural frequency was approximately 120 Hz. Results for both the first and second modes are given in Table 9.

From Table 9 it can be seen that the majority of the damping factors are between 0·0008 (0·08% damping) and 0·005 (0·5% damping). This damping is similar to the measured damping of the thermowells (Table 7 and Figure 6) and unlined stacks (Section 3). Figure 10 shows measured damping of tubes with rolled-in ends (Table 9), thermowells (Table 7) and multi-span tubes which pass through oversized holes in support plates (Pettigrew *et al.* 1986) as a function of natural frequency. For comparison, the damping factor measured by Dockstader (1954) on a 224.5 ft (68·5 m) tall unlined steel exhaust stack are also shown.

Figures 8 and 10 suggest the damping factors for the thermowells and tubes with rolled in ends should fall between 0·0005 and 0·005. Measurements of heat exchanger tubes that pass through oversized holes in intermediate support plates show damping an order of magnitude higher (Pettigrew 1986). Probably, the friction and micromechanical impact of the tube against the oversized hole produced the majority of damping for these tubes, whereas with

TABLE 9
Damping measured on heat exchanger tubes with rolled-in ends

2 in diameter steel tubes		0·75 in diameter copper tubes	
Nat. freq. (Hz)	Damping factor, ζ	Nat. freq. (Hz)	Damping factor, ζ
42·5	0·0004	42·29	0·0008
47	0·0006	42·75	0·0010
47·5	0·0006	43·12	0·0014
35·5	0·0006	41·7	0·0019
61·5	0·0006	41·36	0·0021
45	0·0006	41·3	0·0022
44·5	0·0007	41·39	0·0024
43·25	0·0008	40·91	0·0026
33·5	0·0009	41·64	0·0029
38·5	0·0009	40·73	0·0033
35	0·0009	41·57	0·0037
58·5	0·0010	40·91	0·0041
36	0·0010	40·63	0·0047
32·5	0·0010	41·71	0·0048
48·5	0·0011	41·36	0·0055
53	0·0011	39·58	0·0058
31	0·0011	121·1	0·0007
31	0·0011	120·1	0·0008
42	0·0011	121·9	0·0008
46	0·0012	121·3	0·0009
40	0·0014	118·3	0·0012
53	0·0015	120·2	0·0012
42·5	0·0016	120·8	0·0012
57·5	0·0017	120·6	0·0013
61	0·0017	120·6	0·0015
30	0·0018	122·1	0·0015
50	0·0020	122·6	0·0015
48·5	0·0021	120·2	0·0016
47·5	0·0023	121	0·0016
39·5	0·0030	119·3	0·0017
61	0·0030	122·3	0·0020
50·5	0·0035	121·6	0·0040
39·5	0·0045		
57	0·0048		
59	0·0048		
45·5	0·0065		
42	0·0110		

rolled-in or clamped ends, the damping is provided by the relative small material damping and energy leakage to the drums.

Table 10 suggests guidelines for damping that reflect these measurements. This table is most applicable for low-amplitude vibration of 0·5–1·5 in (12–38 mm) diameter metallic tubes with spans of 10–60 in (250–1500 mm) between supports and support thicknesses between 0·5 and 1·5 tube diameters. For low-amplitude vibration, less than 2% of tube diameter, material damping is low and micro-mechanical dissipative motion at the

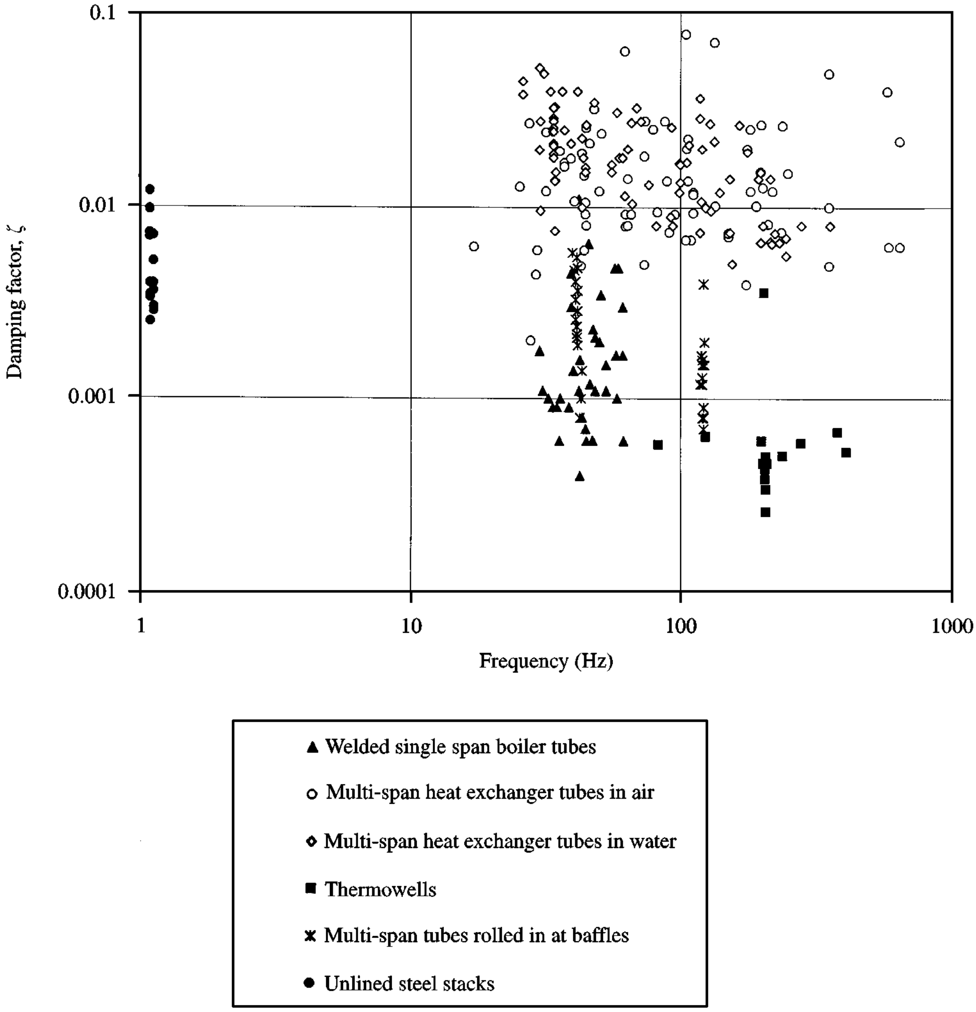

Figure 10. Damping factors of heat exchanger tubes, thermowells, and unlined steel stacks.

TABLE 10
Damping for flow-induced vibration

Description	Fluid about tube	Damping factor, ζ Low	Average	High
Thermowell and single-span tubes with welded or rolled in ends	Liquids and gas	0·0005	0·002	0·005
Multispan heat exchanger tubes passing through oversized holes in plates	Low-density gas	0·008	0·015	0·045
	Water and liquids	0·010	0·020	0·050

supports generally dominates damping. Tubes that are welded or rolled-in at their supports do not benefit from dissipative support interaction and, as a result, their damping is low.

Damping of heat exchanger tubes that pass through oversized holes in support plates will increase with amplitude but there is also increase tube damage at higher amplitudes that can produce fatigue damage or wear. As the tube becomes increasingly active in the hole, damping increases with vibration amplitude until the tube fully traverses the gap between the tube and the hole in the support plate and damping factor peaks at about $\zeta = 0{\cdot}03$ to 0·05, but these larger amplitude vibrations are likely to produce detectable wear between the tube and the support.

The surrounding fluid can contribute to the damping of a tube or thermowell. If damping measurements are made with the tube or thermowell surrounded by a otherwise still fluid, then fluid viscosity will increase damping (Batchelor 1967; Blevins 1990, p. 309) but this is not applicable to vibration in a fluid flow when resonant vortex-induced lift force contributes negative damping.

Tubes passing through oversized holes in support plates will squeeze a film of fluid between the tube and the support during vibration and this is called 'squeeze film damping'. Analytical formulations are available (Pettigrew *et al.* 1986). The data in Section 6 suggest that the average contribution of liquid to damping factor of multispan tubes is between 0·002 and 0·006.

6. DISCUSSION

As noted in Section 2, reduced damping [equation (5)] of 15 or greater will hold amplitude of resonant response to 5% of diameter of less, which is sufficient to ensure that strains in the base of a $U/D \geq 10$ cantilevered thermowell will not fail due to high cycle fatigue;

$$2m(2\pi\zeta)/(\rho D^2) > 15 \quad \text{implies} \quad \rho < 2m(2\pi\zeta)/(15D^2). \tag{11}$$

For heavy-walled thermowells the mass per unit length is approximately $m = (\pi/4)\rho_T D^2$, where ρ_T is the density of the thermowell metal, 0.3 lb/in^3 (8 gm/cm^3) for steel. For a damping factor of 0·0005 this becomes $\rho < 0{\cdot}0003\rho_T$, which implies that in low-density gases, the vortex-induced resonance will not lead to fatigue failure. In particular, for heavy-walled steel thermowells, $\rho_T = 0{\cdot}3$ lb/ft^3 (8310 kg/m^3) fluid densities less than 0·17 lb/ft^3 (2·7 kg/m^3) will not produce damaging resonant vortex-induced vibration. Thus, most low-density gases will not induce damaging resonance of a thermowell.

In high-density gases and liquids, damping will not preclude the possibility of a damaging resonance if the shedding frequency, equation (1), approaches a natural frequency, equation (3). The natural frequencies of the thermowell can be raised slightly by tapering the thermowell, but tapering reduces the diameter of the tip, raising the local shedding frequency. Shortening the thermowell raises its natural frequency. It is common practice to have the thermowell extend to the centreline of piping (Figure 1). This minimizes conduction from the pipe wall to the temperature sensing element and to provide a measurement of the centreline flow. However, because of the high mixing, near-uniform profile in turbulent flow in pipes, and low conductivity of stainless steel and nickel/iron/chromium alloys, acceptable temperature measurement can probably be achieved with a shorter length that gives a higher thermowell natural frequency.

7. CONCLUSIONS

Testing and analysis have been performed for vortex-induced vibration of thermowells. The results are summarized as follows.

(a) Thermowells are prone to vortex-induced resonant vibration when the shedding frequency approaches the natural frequency of a cantilever mode of the thermowell. The resonant displacement and stress are limited by damping.

(b) Measurements of damping of thermowells show that the semi-quartile value (75% of measurements above this value) is 0·05% critical damping and the median value is 0·2% critical damping ($\zeta = 0{\cdot}002$). Damping factors between 0·0005 and 0·002 are recommended for the design of thermowells for flow-induced vibration. These damping factors are well below the damping factors of heat exchanger tubes that pass through oversized holes in the support plates, but they are of the same order as the damping factors of welded steel stacks and heat exchanger tubes which are rolled-in at their supports.

(c) Analysis suggests that even for this low damping, heavy-walled thermowells will not be damaged by resonant vibrations if the fluid has low density. For a typical thermowell, if the fluid density is less than 0·17 lb/ft^3 (2·7 kg/m^3), then even resonant vortex-induced vibration will not be damaging.

ACKNOWLEDGEMENT

The tests of thermowells were made at the Structural Dynamic Research Corporation, San Diego, California by Dan Hensley and Ralph Brillhart. The test thermowells were supplied by Alloy Engineering, Bridgeport Connecticut. The two fixture plates were supplied by the Locke Equipment Company, Olathe, Kansas. The thermocouple insert was supplied by Zeuschel Equipment Company, Lenexa, Kansas.

References

ASME Boiler and Pressure Vessel Code 1995 *Flow-induced vibration of tubes and tube banks*. Section III, Division 1, Appendix N-1300.

ASME Boiler and Pressure Vessel Code 1995 Section III, Division 1, Appendix I, Figure I-9.2.2.

ASME Standard 1993 Steel stacks. STS-1-1992 Para 5.3.2 (c).

ASME Standard 1974 *Temperature measurement*. Performance Test Codes. PTC 19.3-1974 (R1986) Chapter 1, Para. 14.

Bachelor, G. K. 1967 *An Introduction to Fluid Dynamics*. Cambridge: Cambridge University Press, pp. 356–357.

Blevins, R. D. 1979 *Formulas for Natural Frequency and Mode Shape*. Malabar, FL: Kreiger.

Blevins, R. D. 1990 *Flow-Induced Vibration*, 2nd edition, New York: Van Nostrand Reinhold; Reprinted by Malabar, FL: Krieger.

Dockstader, E. A., Swiger, W. F. & Ireland, E. 1954 Resonant vibration of steel stacks. *ASCE Proceedings* **80,** 541-1–541-27.

Pettigrew, M. J., Goyder, H. G. D., Qiao, Z. L. & Axisa, F. 1986a Damping of multi-span heat exchanger tubes. Part 1: in gases. In *Flow-Induced Vibrations-1986* (eds S. S. Chen, J. C. Simonis & Y. S. Shin), PVP-Vol.-104, pp. 81–87, New York: ASME.

Pettigrew, M. J., Rogers, R. J. & Axisa, F. 1986b Damping multi-span heat exchanger tubes. Part 2: in liquids. In *Flow-Induced Vibrations-1986* (eds S. S. Chen, J. C. Simonis & Y. S. Shin), PVP-Vol.-104, pp. 89–98, New York: ASME.

Sarpkaya, T. 1979 Vortex-induced oscillations. *Journal of Applied Mechanics* **46,** 241–258.

Weaver, W., Timoshenko, S. P. & Young, D. H. 1990 *Vibration Problems in Engineering*, 5th ed. New York: John Wiley.

APPENDIX: NOTATION

A_y	transverse zero-to-peak amplitude
C_L	lift coefficient (dimensionless)
D	diameter
E	modulus of elasticity
f	frequency (Hz)
I	moment of inertia
k	spring constant
m	mass per unit length
S	Strouhal number, dimensionless
t	time
U	unsupported span of thermowell
V	flow velocity
x	distance along span
y	dimensionless mode shape
δ_r	reduced damping, equation (5)
ζ	damping ratio, fraction of critical damping
λ	dimensionless natural frequency factor, 1·875 for cantilever in first mode
ν	kinematic viscosity of fluid
ρ	fluid density
ρ_T	density of thermowell material